KTH Challenge 2017

Stockholm, 11th June 2017

Problems

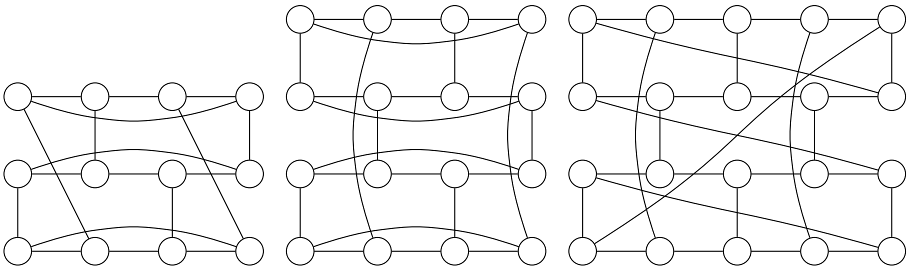
A Saturn Bees
B EvenOdd
C Islands
D House of Cards
E Global Warming
F 3D Printed Statues
G Restaurant Bribes
H Wolf
I Lights Out

This page is intentionally left (almost) blank.

Problem A
Saturn Bees
Problem ID: saturnbees

The Saturn bee (*Apis saturnii*) is quite an interesting species. To begin with, they build their hives in the shape of a ring. More precisely, a beehive is a hexagonal grid, which we represent as a graph where walls are edges and wall joins are vertices. If we flatten each hexagon a bit so that it becomes a 1×2 rectangle, then we can assign an integer coordinate to each vertex so that vertex (i, j) is adjacent to vertices $(i, j-1)$, $(i, j+1)$ and either $(i+1, j)$ if $i+j$ is odd, or $(i-1, j)$ if $i+j$ is even.

To make an $n \times m$ grid into a ring the edges wrap around. So, if n and m are even, an edge with endpoint (n, j) will end at $(0, j)$ instead, and an edge with endpoint (i, m) will end at $(i, 0)$. If either coordinate is odd the bees need to twist the grid so that both sides will match: if n is odd then (n, j) becomes $(0, j+1)$, and if m is odd then (i, m) becomes $(i+1, 0)$. The swarm mind is aware of the handshaking lemma and does not try to build beehives where both n and m are odd. See Figure A.1 for a few examples of beehives.



(a) $H_{3,4}$ **(b)** $H_{4,4}$ **(c)** $H_{4,5}$

Figure A.1: Example beehives

Another outstanding fact about Saturn bees is how they guard their hive. Each soldier bee sits on top of a vertex and its task is to control that vertex and the 3 adjacent vertices. The swarm mind is aware that $nm/4$ bees are required for this, hence this is the number of soldiers in the swarm, but unfortunately some beehives are turning tricky to guard and the Saturn bees refuse to live there.

Your task is to determine whether a beehive is a suitable home for a swarm.

Input

The first line of input contains two integers, n and m ($2 \leq m, n \leq 10\,000$, m or n even).

Output

Output "`possible`" if $nm/4$ bees can guard an $n \times m$ beehive, and "`impossible`" otherwise.

Sample Input 1	Sample Output 1
`4 6`	`impossible`

Sample Input 2	Sample Output 2
`6 4`	`possible`

Problem B
EvenOdd
Problem ID: evenodd

Consider the following function $f(X)$, which takes a single positive integer as argument, and returns an integer.

```
function f(X):
    iterations := 0
    while X is not 1:
        if X is even:
            divide X by 2
        else:
            add 1 to X
        add 1 to iterations
    return iterations
```

It can be shown that for any positive integer X, this function terminates. Given an interval $[L, R]$, compute the sum

$$S = f(L) + f(L+1) + \cdots + f(R-1) + f(R)\ .$$

Input

The first and only line of input contains two integers L and R ($1 \leq L \leq R \leq 10^{18}$).

Output

Output the result S modulo the prime $10^9 + 7$.

Sample Input 1	Sample Output 1
1 127	1083

Sample Input 2	Sample Output 2
74 74	11

This page is intentionally left (almost) blank.

Problem C
Islands
Problem ID: islands

Weenesia is an archipelago of perfectly circular islands in the 2D plane. Many islands have palm trees growing in them, perfectly straight up, so we can represent a tree by a 2D point and a height.

We want to build a courier system in the islands, i.e., we want to be able to move any object from any land point to any other land point. It is possible to move an object within an island without restrictions. It is also possible to climb to a palm tree and throw the object to a distance proportional to the height of the palm tree (or less), and possibly in a different island.

Unfortunately, this may not be enough to reach our goal, so we might want to build a tunnel between two islands. The tunnel connects two points on two islands and may cross under both the sea and other islands. Each of the two entrances of a tunnel must be at least 1 meter away from the sea to avoid flooding.

Your task is to find the minimum length of a tunnel such that a courier system is possible.

Input

The first line contains three integers n, m, and k ($1 \leq n \leq 5\,000$, $0 \leq m \leq 10\,000$, $1 \leq k \leq 1\,000$), the number of islands and palm trees respectively, and the ratio between object throwing range and palm height.

Each of the next n lines contains three integers x, y, and r ($|x|, |y| \leq 10^6$, $100 \leq r \leq 10^6$), the center and radius of an island, in centimetres. Each of the next m lines contains three integers x, y, h ($|x|, |y| \leq 10^6$, $1 \leq h \leq 10^4$), the center and height of a palm tree, in centimetres.

No two islands intersect. Each palm tree lies strictly inside an island. No two palm trees grow in the same spot.

Output

Output the minimum length of a tunnel in centimetres, 0 if no tunnel is needed, or "`impossible`" if no such tunnel exists. Answers with an absolute or relative precision up to 10^{-6} will be accepted.

Sample Input 1	Sample Output 1
3 2 3 0 0 400 1000 0 400 2000 0 400 300 0 150 1300 0 150	1400

Sample Input 2	Sample Output 2
3 2 2 0 0 400 1000 0 400 2000 0 400 300 0 100 1300 0 100	impossible

Problem D
House of Cards
Problem ID: houseofcards

Brian and Susan are old friends, and they always dare each other to do reckless things. Recently Brian had the audacity to take the bottom right exit out of their annual maze race, instead of the usual top left one. In order to trump this, Susan needs to think big. She will build a house of cards so big that, should it topple over, the entire country would be buried by cards. It's going to be huge!

Picture by Liftarn on Wikipedia, public domain

The house will have a triangular shape. The illustration to the right shows a house of height 6, and Figure D.1 shows a schematic figure of a house of height 5.

For aesthetic reasons, the cards used to build the tower should feature each of the four suits (clubs, diamonds, hearts, spades) equally often. Depending on the height of the tower, this may or may not be possible. Given a lower bound h_0 on the height of the tower, what is the smallest possible height $h \geq h_0$ such that it is possible to build the tower?

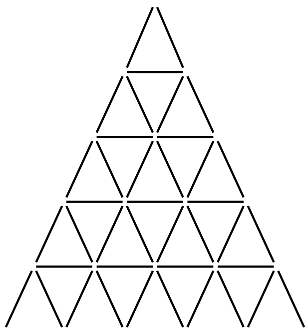


Figure D.1: A house of height 5 uses 40 cards.

Input

A single integer $1 \leq h_0 \leq 10^{1000}$, the minimum height of the tower.

Output

An integer, the smallest $h \geq h_0$ such that it is possible to build a tower of height h.

Sample Input 1	Sample Output 1
2	5

Sample Input 2	Sample Output 2
42	45

This page is intentionally left (almost) blank.

Problem E
Global Warming
Problem ID: globalwarming

This is a very exciting week for John. The reason is that, as a middle school teacher, he has been asked to dedicate the entire week to teaching his class of n students about the cause and effect of global warming. As John is very passionate about his planet, he's going to spend extra time and effort to make this week memorable and rewarding for the students. Towards that, one of the things he wants to ask them to do is to prepare, as homework, presentations about global warming. To make this a little easier for them, as well as more fun, he has asked them to do this in groups of two.

Of course arranging the students into groups comes with the usual headache, namely that only friends are willing to work together. Luckily the students in his class are a friendly bunch. In particular, if p, q and r are three distinct students, and p and q are friends, and q and r are friends, then p and r are also friends. But John now realizes the irony in asking his students to work at home in groups, as students may have to travel to meet their group partner, which may emit greenhouse gases such as carbon dioxide, depending on their mode of transportation. In the spirit of this week's topic, John asked all the students in his class to calculate, for each of their friends, how much carbon dioxide would be emitted if they were to meet up with the respective friend.

Using this information, can you help John figure out what is the minimum total amount of carbon dioxide that will be emitted if he arranges the groups optimally, or determine that it's not possible to arrange all the students into groups of two friends?

Input

The first line contains two integers n and m ($1 \leq n \leq 200$, $0 \leq m \leq 250$), the number of students in John's class, and the total number of pairs of friends in the class. As John is bad with names, he has given each of his students a distinct integer identifier between 1 and n.

Each of the next m lines contains three integers p, q and c ($1 \leq p, q \leq n$, $0 \leq c \leq 10^6$), the identifiers of two distinct students that are friends, and how many grams of carbon dioxide would be emitted if they were in a group together, and thus had to meet. Each pair of friends is listed exactly once in the input.

Output

Output the minimum total amount of carbon dioxide, in grams, that would be emitted if John arranges all students optimally into groups of two friends, or "`impossible`" if there is no way to arrange the students into groups in that way.

Sample Input 1	Sample Output 1
5 4 3 1 375 2 5 283 1 4 716 3 4 98	impossible

Sample Input 2

```
6 7
5 6 600
2 5 200
3 5 400
6 3 500
1 4 300
3 2 400
6 2 200
```

Sample Output 2

```
900
```

Problem F
3D Printed Statues
Problem ID: 3dprinter

You have a single 3D printer, and would like to use it to produce n statues. However, printing the statues one by one on the 3D printer takes a long time, so it may be more time-efficient to first use the 3D printer to print a new printer. That new printer may then in turn be used to print statues or even more printers. Print jobs take a full day, and every day you can choose for each printer in your possession to have it print a statue, or to have it 3D print a new printer (which becomes available for use the next day).

What is the minimum possible number of days needed to print at least n statues?

Input

The input contains a single integer n ($1 \leq n \leq 10\,000$), the number of statues you need to print.

Output

Output a single integer, the minimum number of days needed to print at least n statues.

Sample Input 1	Sample Output 1
1	1

Sample Input 2	Sample Output 2
5	4

This page is intentionally left (almost) blank.

Problem G
Restaurant Bribes
Problem ID: restaurantbribes

Picture by Desiree Williams, cc by-nd

Several participants in programming competitions end up founding startup companies. Two of them created Eeet, a social restaurant rating system. Each member of Eeet can rate a restaurant with an integer score from 0 to 10 or not give a score at all. The score that a person sees is the average of ratings from the friends who gave one. In particular, a person giving a score of 0 will in general decrease the rating that their friends see, whereas not giving a score at all will not affect it. If none of the friends of a user has given a rating to a restaurant, then the restaurant does not show up to that user.

Two other past contestants have opened a restaurant, The Smelly Fish, but they are bewildered as to why nobody is dining at their establishment. Through experimentation they found out that each person is willing to come once to the restaurant (for some inexplicable reason no customer ever returns) and spend $100y$ SEK, where y is the rating they see on Eeet. Everybody pays by card, and money transactions are not rounded.

However, they did not get any ratings yet, and all the fresh produce they bought risks going past its prime. Hence they selected some people to bribe so that they will leave them a rating in Eeet. For each person, we know that they will give The Smelly Fish a rating according to the function $\lfloor \sqrt{x}/a \rfloor$, where a is a constant depending on the person, and x is the amount of the bribe in SEK. Note that a person bribed with 0 SEK will leave a rating of 0 (instead of not giving a rating at all), that the maximum rating is 10, and that a person that received a bribe knows about the scheme and will not go to the restaurant.

Your task is to maximize the profit the restaurant makes, i.e., the income from customers minus the money spent on bribes.

Input

The first line contains three integers n, m, and k, ($1 \leq m \leq 10^5$, $0 \leq n \leq 10^5$, $0 \leq k \leq n$), the number of people in Eeet, the number of friendship relations, and the number of people we are going to bribe.

Each of the next m lines contains two integers, a and b ($1 \leq a, b \leq n$), the ids of a pair of friends. No unordered pair $\{a, b\}$ appears more than once.

Each of the next k lines contains two integers i ($1 \leq i \leq n$), the id of a person we are going to bribe, and the parameter a_i ($1 \leq a_i \leq 1\,000$). No id appears more than once.

Output

Output one line with the maximum profit the restaurant can make. Answers with an absolute or relative precision up to 10^{-6} will be accepted.

Sample Input 1	Sample Output 1
10 1 1 1 2 1 3	276

Problem H
Wolf
Problem ID: wolf

In the card game Wolf (a variant of the famous card games *War* and *Svälta räv*), two players both have a set of cards, which together form a normal deck of playing cards. Such a deck contains 52 cards, with each card being a unique combination of one of 4 suits and 13 ranks.

During a turn, each player picks the topmost card from their piles, and shows it. This process is repeated until the two cards have the same suit. Then, the player with the highest rank receives all the cards, which are shuffled into the deck. The turn then ends. The turn also ends if any player, when they are suppoed to pick a card from their pile, has an empty pile. That player will then lose the game. This means that the game can be tied, if the two players both try to pick a new card simultaneously.

Your opponent is currently taking a bathroom break, so you have the opportunity to reshuffle your decks. You cannot exchange any cards, since your opponent knows the set of cards in their deck. Is it possible to reshuffle the deck, so that you win during the next turn?

Input

The first line of input contains the number of cards in your hand ($0 \leq n \leq 52$). Each of the next n lines contains a card specified by a number from 1 to 13 and one of the letters 'C', 'D', 'H', 'S', separated by a space.

The letter of a card specifies its suit (either clubs, diamonds, hearts or spades, respectively). The number of a card specifies the rank of the card. For simplicity, aces are represented as a 1, and jacks, queens and kings are represented as the numbers 11, 12 and 13. For the purpose of this problem, we consider a rank a to be lower than a rank b if the number of rank a is lower than the number of rank b (meaning aces are of the lowest rank).

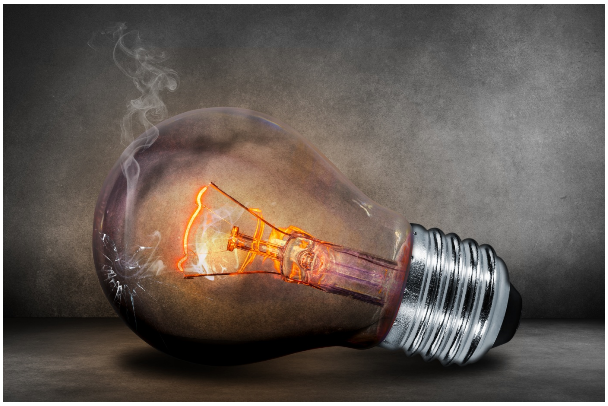
Output

Output "`Possible`" if it is possible to reshuffle the decks, and "`Impossible`" if it is not.

Sample Input 1	Sample Output 1
28 1 C 2 C 3 C 4 C 5 C 6 C 7 C 1 D 2 D 3 D 4 D 5 D 6 D 7 D 1 H 2 H 3 H 4 H 5 H 6 H 7 H 1 S 2 S 3 S 4 S 5 S 6 S 7 S	possible

Sample Input 2	Sample Output 2
0	impossible

Problem I
Lights Out
Problem ID: lightsout



Poor Eve, she is almost always the last one at work. Most unfortunately, she is very afraid of the dark, and the company rules dictates the last one leaving the office is obliged to make sure all the lamps at the whole office are off. Her sloppy colleagues sometimes forget to turn their lamps off, more often then not to be honest, but to be fair it is not as trivial as you might think. You see, at Eve's workplace, each room has exactly one lamp but may have several light switches. The thing is though, that unlike a traditional light switch, these switches toggle a subset of all the lamps at the office. Each switch inverts the light status, either from lit to turned off or the other way around, for a fixed set of lamps depending on the switch. It may even be that the lamp in the room is not effected by any of its switches, or that there are no switches in the room at all.

Eve is a creature of habit and wants to take a fixed route out of the office each evening. At the same time, the set of lamps to turn off may be different on different days, so she has to plan for all eventualities. In other words, Eve wants a route through the office such that, for any possible configuration of the lamps, there is some combination of the light switches in the rooms Eve moves through that will allow her to turn off that configuration of lamps.

Note that it may be the case that some configurations of lamps are impossible to turn off (for instance, in Sample Input 1, it would be impossible to turn off only lamp 0), but Eve doesn't have to worry about such configurations of lamps (because they are also impossible to turn on). The route only has to let her turn off configurations which are actually possible to turn off. Eve is fine with passing through a room even if its lamp is turned off, for instance it is OK if the last lamp in the office is turned off somewhere in the middle of the route even though it means walking through a few unlit rooms at the end.

Input

The first line of input contains three positive integers, n, m, and l ($2 \leq n \leq 20$, $1 \leq m \leq 190$, $1 \leq l \leq 100$), where n is the number of rooms in the office, m is the number of connections between rooms, and l is the number of switches. Next follow m lines, each describing a pair of adjacent rooms containing two room numbers $a \neq b$, meaning you can enter room b from room a and vice versa. No unordered pair $\{a, b\}$ appears more than once.

Then follow l lines, each describing a light switch. Each such line starts with a room number telling which room the light switch is in. The second integer on the line $p > 0$ gives the number of lamps that are toggled by the switch. The remainder of the line contains p room numbers. You can assume that no two room identifiers are identical in a switch's toggle list.

The rooms are numbered 0 through $n - 1$. The room from which Eve leave's the office is number 0, and Eve's room (where she starts) is number 1. You may assume that it is possible to reach any room from room 1.

Output

Output one line with the minimum number of rooms on a path from Eve's room to the entrance room (counting both endpoints) including a set of switches making it possible to turn off any possible subset of lamps lit. Note that she might need to visit a room multiple times, in which case each visit should be counted.

Sample Input 1	Sample Output 1
6 5 7 0 2 2 3 3 4 3 5 1 2 1 2 0 1 1 2 1 2 2 2 2 3 3 2 3 4 4 2 4 5 4 2 0 5 5 2 0 3	7