GCPC 2020
Presentation of solutions

Jury and Testers

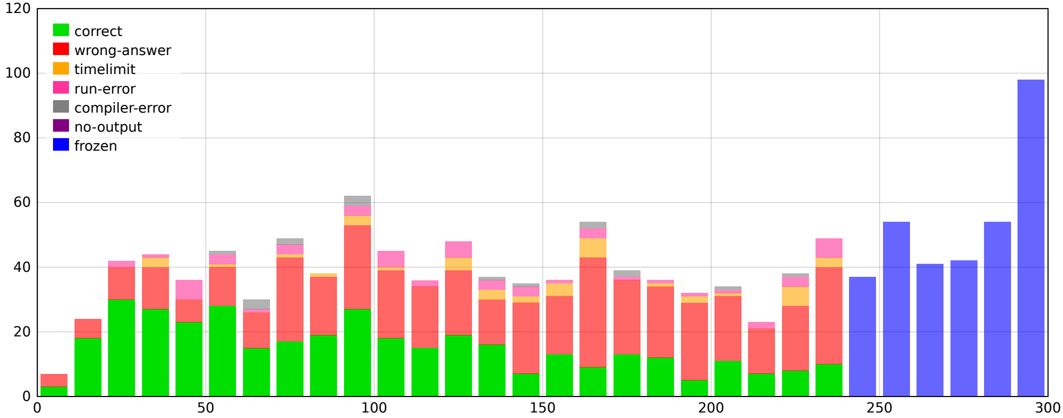
Thanks to the jury:

- Michael Baer (FAU)
- Julian Baldus (UdS)
- Gregor Behnke (Freiburg)
- Sandro Esquivel (CAU)
- Maximilian Fichtl (TUM)
- Nathan Maier (Ulm)
- Tobias Meggendorfer (TUM)
- Philipp Reger (FAU)
- Gregor Schwarz (TUM)
- Paul Wild (FAU)

Thanks to our test readers:

- Gregor Matl (TUM)
- Marcel Wienöbst (Lübeck)

Statistics



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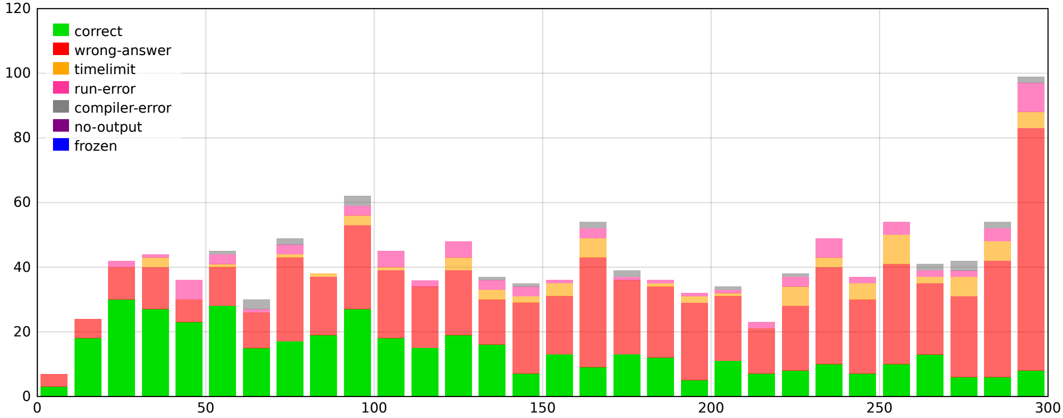

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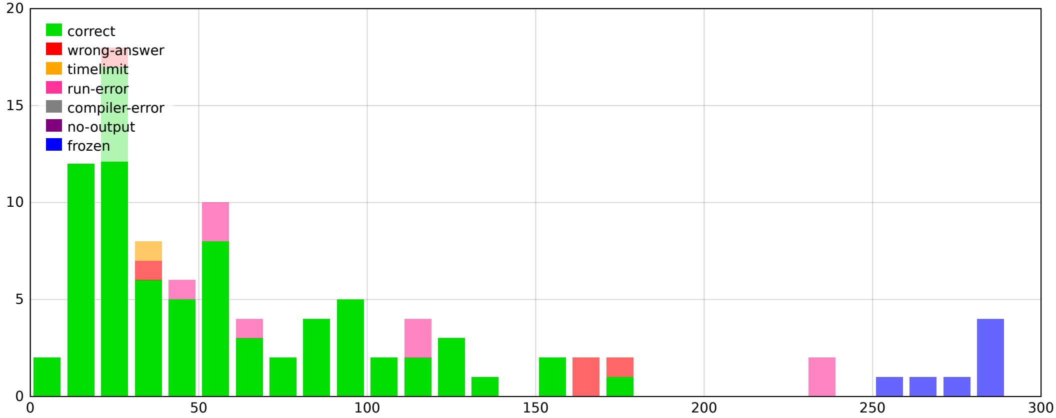
F – Flip Flow



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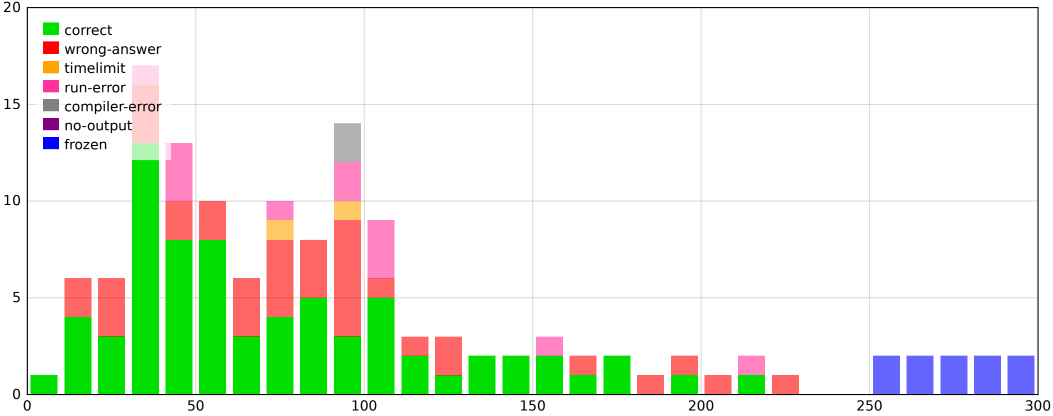
Solution

The process finishes after at most 10^6 seconds, so simulate step by step:

```
int upper = 0, lower = s;
for (int k = 0; k < t; k++) {
    if (flip[k]) swap(upper,lower);
    if (upper > 0) upper--, lower++;
}
```

Can also be solved in $\mathcal{O}(n)$, where n is the number of flips.

A – Adolescent Architecture



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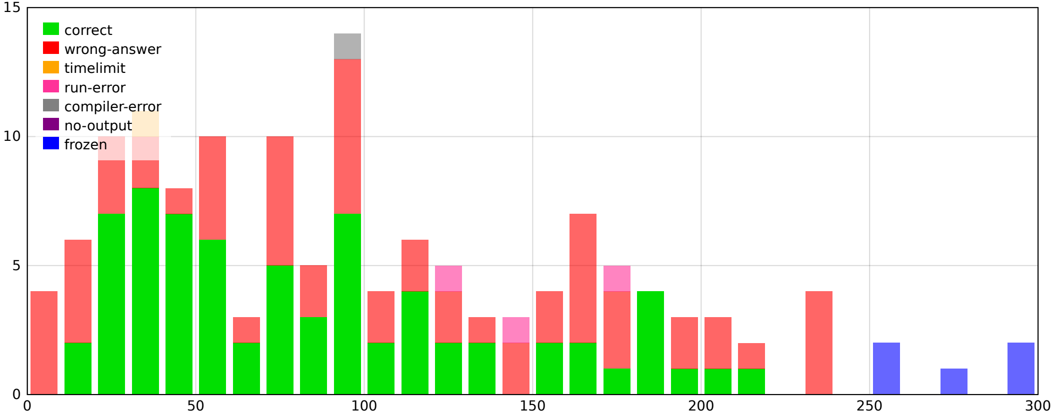
- If a solution exists, the footprint area of the blocks must always decrease towards the top of the tower.
- Sort the blocks by area and check for each adjacent pair if one fits inside the other.
- There is a simple formula for each of the four cases.

M – Mixtape Management



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Given a permutation $p_1, \ldots, p_n$, find a sequence of positive integers $a_1, \ldots, a_n$ where $\text{str}(a_1) < \cdots < \text{str}(a_n)$ lexicographically and $a_{p_1} < \cdots < a_{p_n}$ by value.

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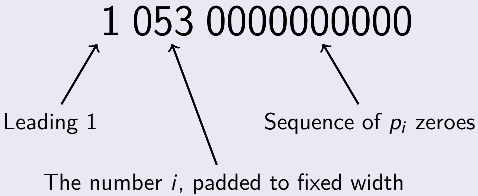
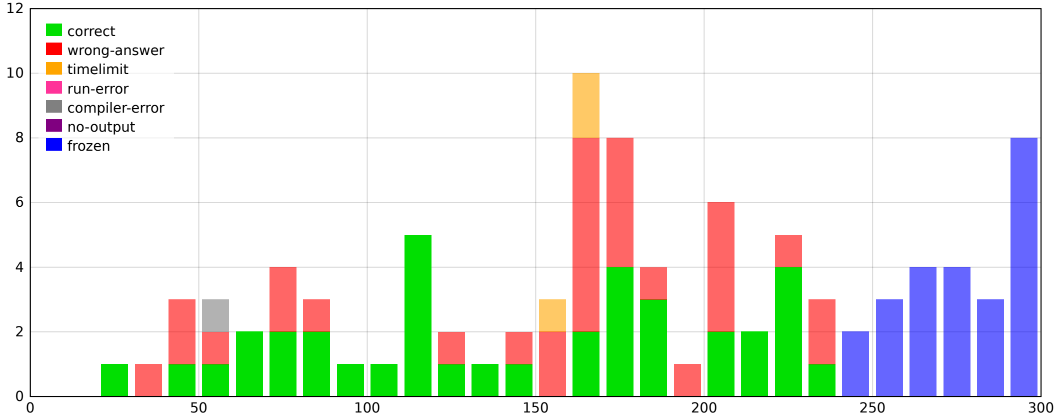
Solution

- Compose every number from three parts:



- This guarantees that all numbers are valid and the sorting is correct in both cases.

C – Confined Catching



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Solution

Obviously, you have to move your pieces towards the AI's.

C – Confined Catching

Solution cont.

However, just reducing the distance in all your turns may not be enough, as the AI may be able to keep fleeing forever.

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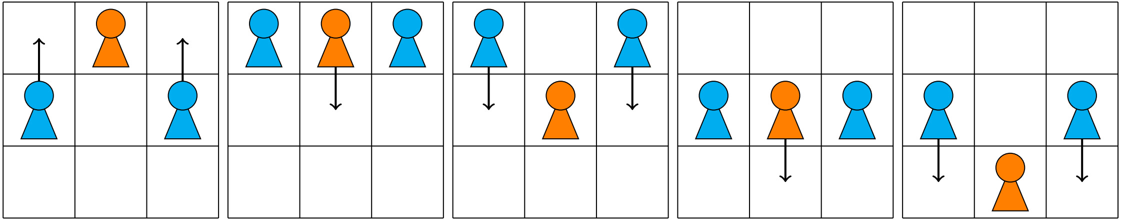

C – Confined Catching

Solution cont.

The crucial strategy is to have your two pieces behave slightly differently:

- For your first piece, if it could move along either axis to get closer to the opponent, move along the y axis first.
- For your second piece, prioritize the x axis.

Eventually, the AI will be forced into a corner (or even lose before that), with nowhere left to run.

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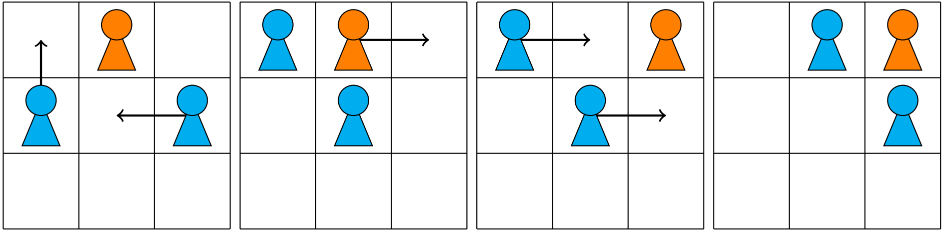
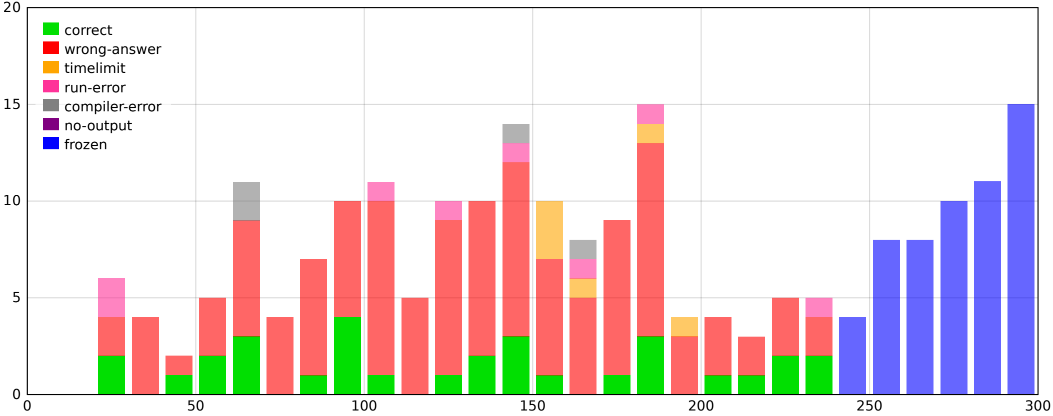


B – Bookshelf Building



B – Bookshelf Building

Problem

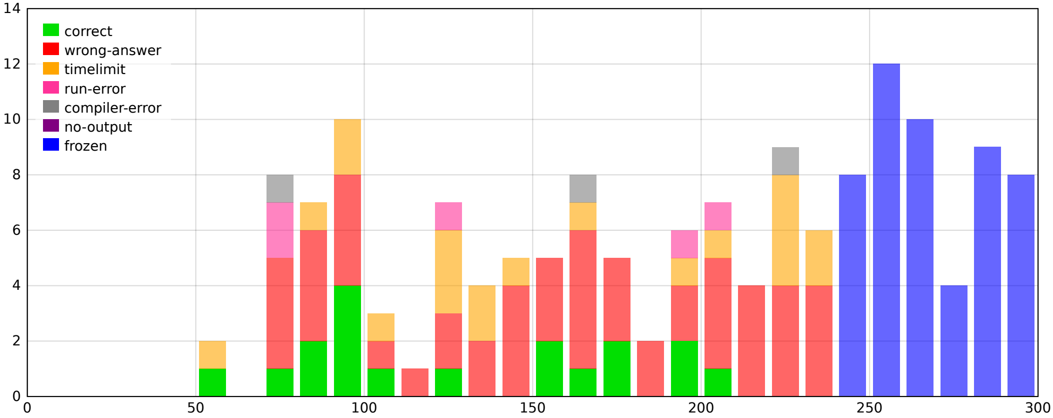
Given n books of different widths and heights, can you fit them into a rectangular bookshelf using at most one separating board?

B – Bookshelf Building

Solution

- Place the tallest book in the bottom left corner of the shelf.
- Install the board at the height of the tallest book.
- Greedily place all books in the lower section of the shelf that do not fit in the upper section.
- For the remaining capacity in the lower section, solve a knapsack problem – the more you fit into the lower section, the more space you have left in the upper section.
- Place all remaining books in the upper section.
- Special case: install no board if tallest book has height y
- Complexity: $\mathcal{O}(n\,x)$

G – Gravity Grid



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Alice and Bob are playing a version of Connect Four on an $h \times w$ board where the goal is to complete a row of k tiles. Given a log of their moves, determine the winner.

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- Time and space complexity: $\mathcal{O}(h \cdot w)$.

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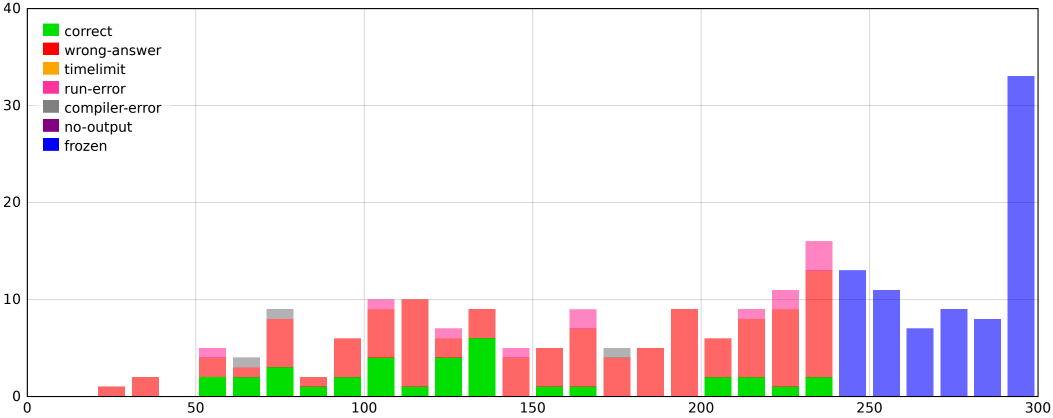
- After each drop, it is enough to update the values in the current cell and the opposite cell of the run in each direction.
- Time and space complexity: $\mathcal{O}(h \cdot w)$.
- Several other solutions are possible, for instance using binary search and a two-pointer method or using a monotone queue.

K – Knightly Knowledge



K – Knightly Knowledge

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Given are the coordinates of *monuments* and *churches*. Churches with ≥ 2 mon. in their row / col are *mighty*. Place one monument to maximize the number of churches that are turned mighty.

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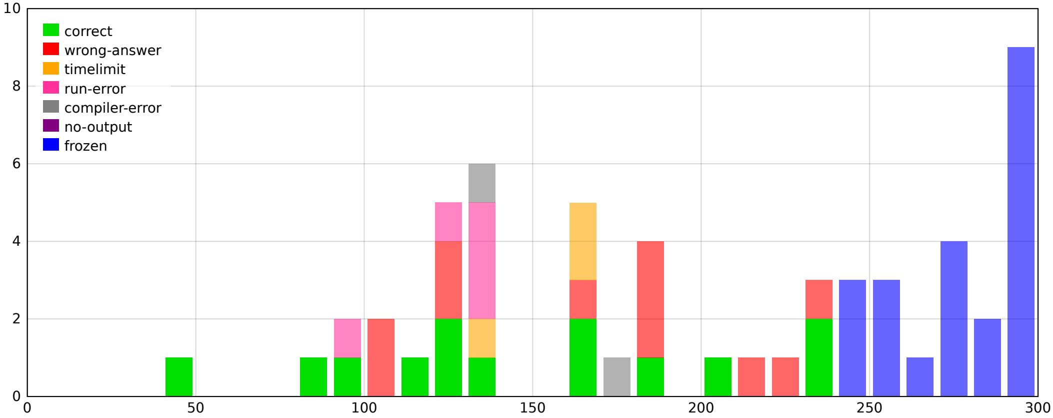
Solution $O((m+c)^2)$

- Count monuments and ordinary churches per row / column.
- Iterate over all reasonable locations to find the best spot.
- *Reasonable:* At least on church or monument in the same row and column.
- Do *not* count a church at the intersection twice.

Solution $O(m+c)$

- As above, but only check best row / col. They are either optimal or off by one.
- $\rightsquigarrow$ No other intersection can be better.
- Each non-optimal intersection has a church at the spot $\rightsquigarrow$ at most c.

D – Decorative Dominoes



D – Decorative Dominoes

Problem

Given an arrangement of dominoes without numbers, assign numbers to both halves of each domino such that

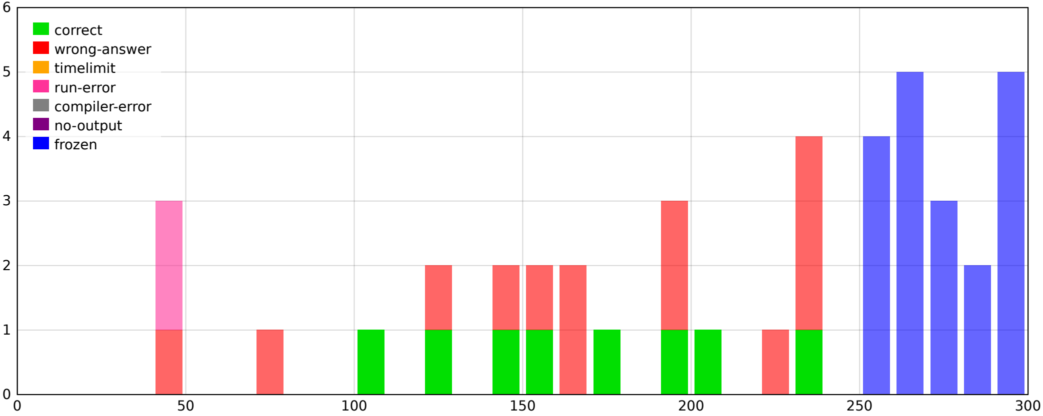
- each domino half is adjacent to a half of another domino with the same number on it;
- all numbers appear at most twice among all dominoes.

D – Decorative Dominoes

Solution

- Transform the problem into a graph where the nodes are the domino halves and the edges exist between adjacent halves belonging to different dominoes.
- The graph is bipartite: Imagine a large black and white checkered board over the coordinate grid. Connected nodes must have different colors.
- Find a perfect bipartite matching on this graph.
- Matched nodes are assigned the same number.
- Every valid numbering corresponds to a perfect matching.
  So whenever a solution exists, this algorithm will find one.
- Complexity: $\mathcal{O}(n^2)$

I – Impressive Integers



I – Impressive Integers

Problem

For a given integer n, determine if there exist integers a, b, and c such that an equilateral triangle with side length c can be tiled with exactly n triangles with side lengths a or b.

If possible, output a valid tiling.

I – Impressive Integers

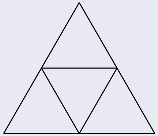
Solution

- By trying out small numbers one can find that it is impossible for $n = 2, 3, 5$.
- For all other $n > 0$ a valid tiling can be found as follows:

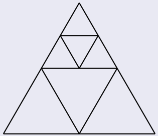
  $n = 1$ $a = b = c$

  $n > 2$ is even Use pattern 1a with $n - 1$ triangles in the bottom row.

  $n > 5$ is odd Use pattern 1b with $n - 4$ triangles in the bottom row.



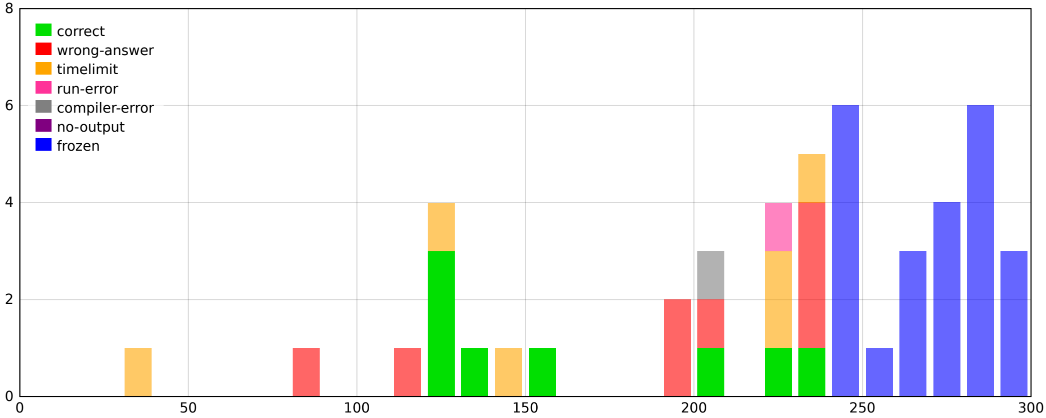
(a) Even pattern.



(b) Odd pattern.

- Complexity: $\mathcal{O}(n)$

L – Lexicographical Lecturing



L – Lexicographical Lecturing

Problem

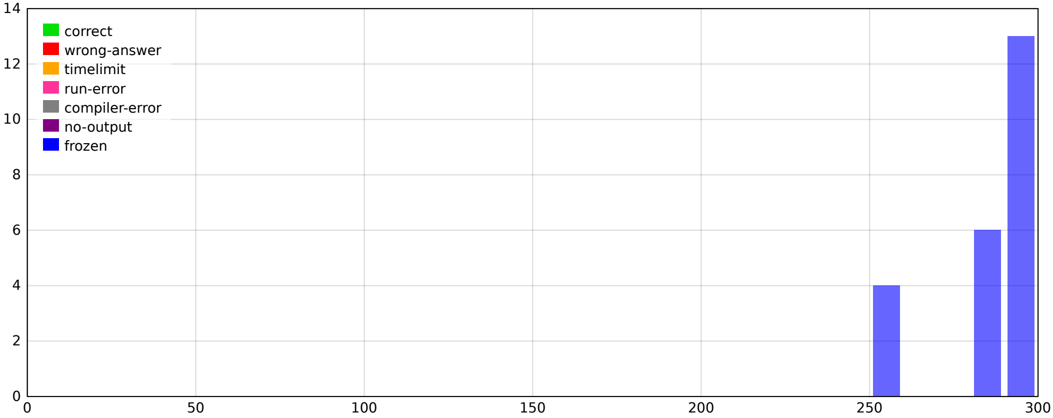
Find indices $i < j$ describing minimal-length substrings, such that the order with respect to the substring is equal the the original order.

L – Lexicographical Lecturing

Solution

- If any two subsequent strings s_k, s_{k+1} are ordered correctly w.r.t. interval (i, j), then all strings can be ordered correctly w.r.t. (i, j).
- Consider all subsequent strings s_k, s_{k+1} one after another.
- For each index i, let α_{ki} be the smallest index such that s_k and s_{k+1} are sorted correctly w.r.t. interval (i, α_{ki}).
  If no such index exists, set $\alpha_{ki} = \infty$.
- Determining all α_{ki} for two subsequent strings can be done in $\mathcal{O}(\ell)$ using dynamic programming.
- For each index i, determine the maximum α_{ki} over all k.
- Output the shortest interval among all $(i, \max_k\{\alpha_{ki}\})$.
- Complexity: $\mathcal{O}(n\,\ell)$

J – Jeopardised Journey



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Problem

Given points (glades) and circles (hills). You can go from point A to B iff the direct line between them does not intersect a circle or any other point.
Opponent selects (unknown) one point to block. Determine for a starting point, which other points can be reached no matter which point is blocked by the opponent.

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Solution

Two part problem:

- Geometry: Determine for which points A and B there is no circle/other point between them and build a graph.
- Graph: Find all nodes of the graph with two fully disjunct paths to node 0.

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Solution: Geometry

1000 points and 1000 circles $\Rightarrow$ We can't test all pairs of points (would be $\mathcal{O}(n^3)$).

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Angular Sorting per point.

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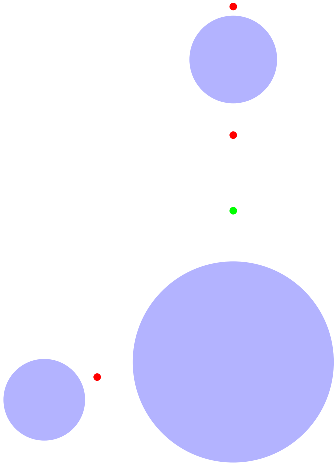

- Consider one point (green).

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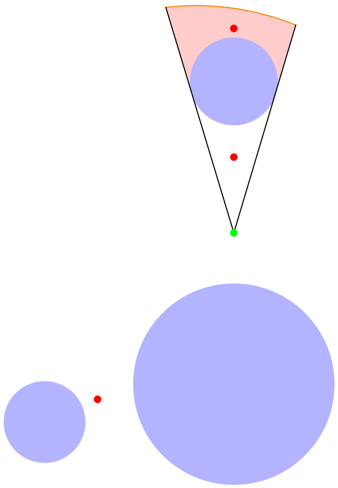
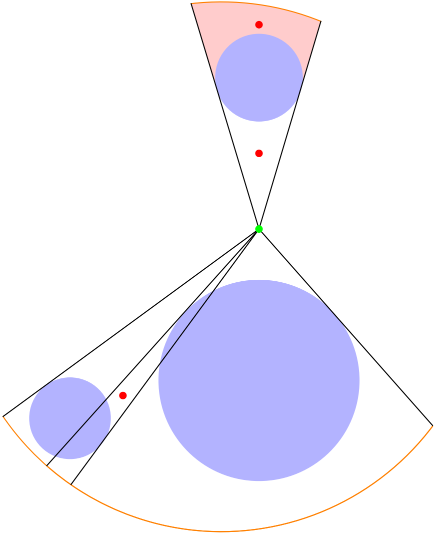


- Consider one point (green).
- A Circle "excludes" all points behind it.



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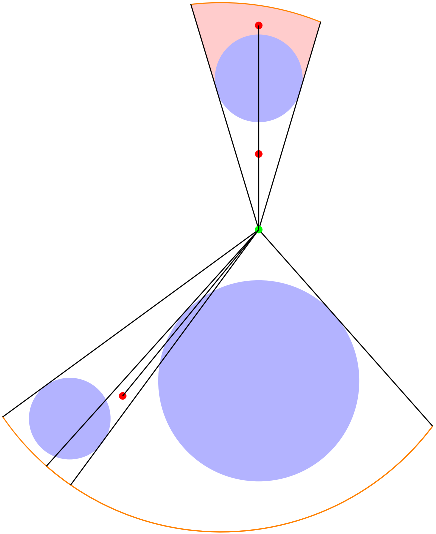

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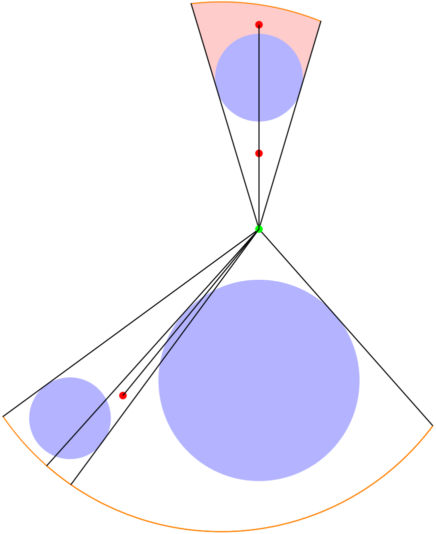

- Consider one point (green).
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- Compute angles between the current point and all other points.
- Sort these ≤ 2999 events (start and end per circle and points)

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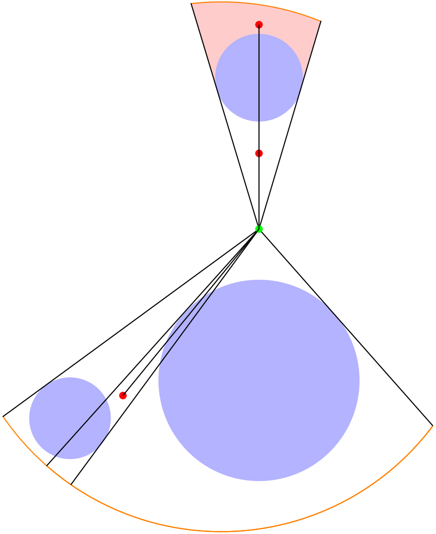

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  - Maintain set S of currently started, but not finished circles (with the distance of the tangent points)
  - If next event is

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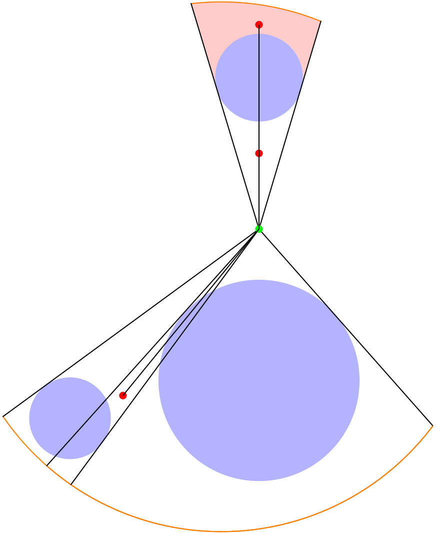

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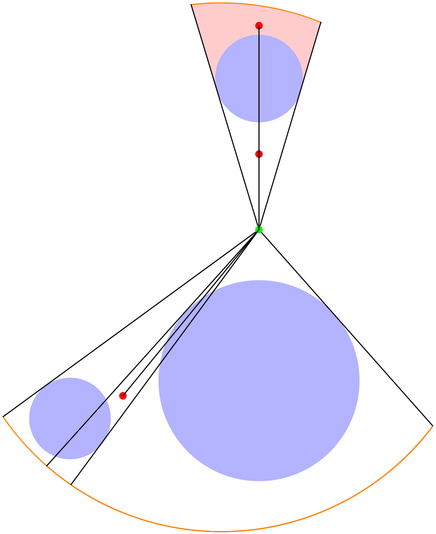

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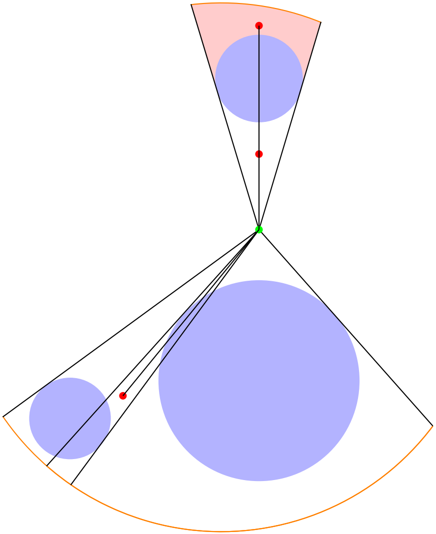

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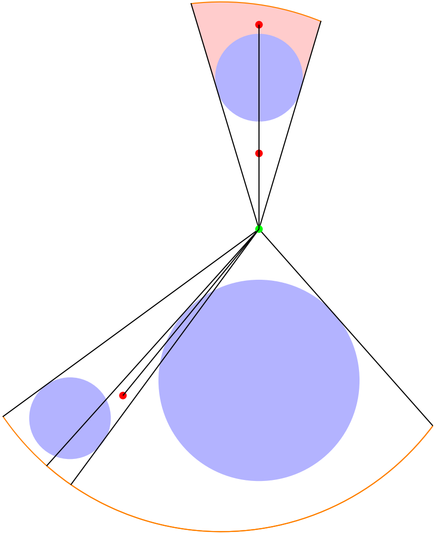

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- Runtime: $\mathcal{O}(n \log n)$

J – Jeopardised Journey

Solution: Geometry Pitfalls

Implementation has a lot of pitfalls

- Multiple events with same angle: end circle – points in increasing distance – begin circle
- For multiple points with same angle: add edge only to the first one.
- Circles that intersect with the $(+, 0)$ axis have $\alpha_{begin} > \alpha_{end}$.
  $\Rightarrow$ split into two along angle 0.

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Solution: Graph

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Try every blocked vertex. Do DFS from node 0.
Count how often each vertex is reached. If it is reached in $|V| - 2$ DFSs (it is blocked once) then it is safe.
Graph contains n^2 edges, so this would have runtime n^3 (too slow).

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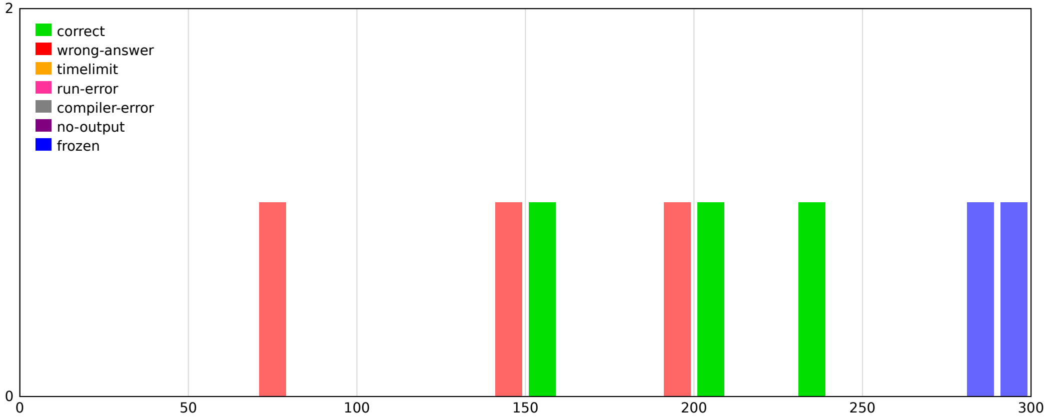
Better solution: Articulation nodes (can be determined in linear time using DFS).
A node is safe if it is reachable from 0 without traversing an articulation node.
$2\times$ DFS $\Rightarrow O(n^2)$

E – Exhausting Errands



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Problem

Given n pairs of integers $(a_1, b_1), \ldots, (a_n, b_n)$, find the length of the shortest 1D route visiting all positions a_i, b_i subject to the constraint that a_i is visited before b_i. The route can start at any a_i and finish at any b_i.

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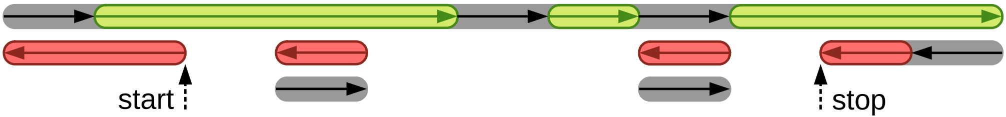
Solution

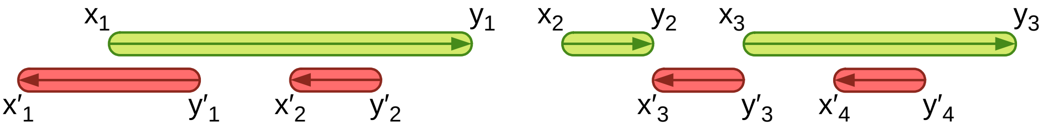
Idea: Complete all “forward“ errands during one left-to-right run, complete “backward” errands either before, after or during this run.



E – Exhausting Errands

Solution: Preparation

- Partition pairs into "forward" pairs ($a_i \leq b_i$) and "backward" pairs ($a_i > b_i$).
- Sort pairs within each partition ascending by start position.
- Merge all overlapping or adjoint pairs within each partition.
- Denote resulting m "forward" pairs as $(x_1, y_1), \ldots, (x_m, y_m)$ and m' "backward" pairs as $(x'_1, y'_1), \ldots, (x'_{m'}, y'_{m'})$.
- Note that $x_1 \leq y_1 < \ldots < x_m \leq y_m$ and $y'_1 < x'_1 < \ldots < y'_{m'} < x'_{m'}$.
- Complexity: $\mathcal{O}(n \log(n))$ for sorting, $\mathcal{O}(n)$ for merging



E – Exhausting Errands

Solution: Special Case 1

- Assume that $m' = 0$. Then the trivial solution is to start at x_1 and finish at y_m (single "forward pass"). The resulting distance is $y_m - x_1$.

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Solution: Special Case 2

- Assume that $m' = 1$. Then we have three options:
  - Go from x'_1 to y'_1 before the forward pass and proceed to x_1 afterwards. The extra distance is $x'_1 - y'_1 + |x_1 - y'_1|$.
  - Stop at x'_1 during the forward pass, go to y'_i and return to x'_i (only applicable if $x_1 < x'_1 < y_m$). The extra distance is $2(x'_1 - y'_1)$.
  - Process to x'_1 and y'_1 after the forward pass. The extra distance is $x'_1 - y'_1 + |x'_1 - y_m|$.

E – Exhausting Errands

Solution: General Case

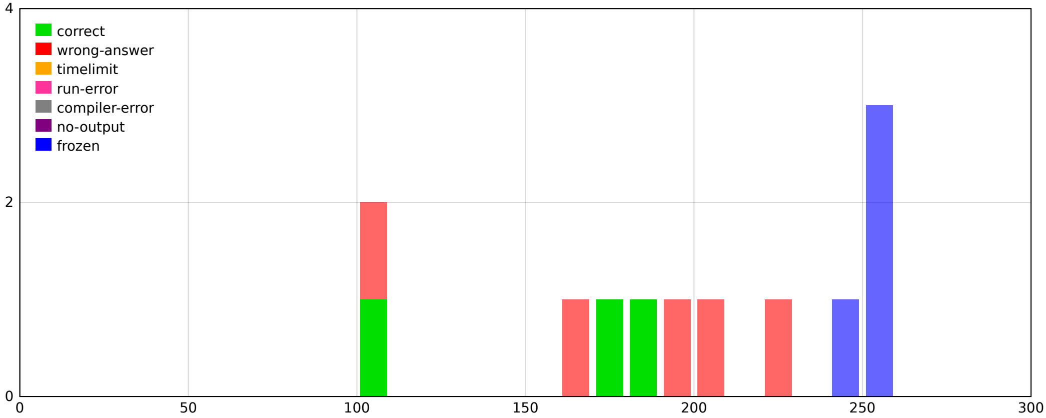
- Assume that $m' > 1$. Now we can complete the first s backward errands before the forward pass, the last t after and the remaining $m' - s - t$ backward errands during the forward pass $(0 \leq s \leq m', 0 \leq t \leq m' - s)$:
  - The extra distance for the first part is $x'_s - y'_1 + |x_1 - y'_1|$.
  - The extra distance for the inbetween part is $\sum_{i=s+1}^{m'-t} 2(x'_i - y'_i)$.
  - The extra distance for the last part is $x'_{m'} - y'_{m'-t+1} + |x'_{m'} - y_m|$.
- Check the total distance for all feasible s, t and pick the minimal solution.
- Complexity: $\mathcal{O}(n^2)$

E – Exhausting Errands

Solution: Caveats

- Solve also the mirrored problem, *i. e.*, for errands $(-a_1, -b_1), \ldots, (-a_n, -b_n)$, and pick its solution if better.
- Do not evaluate special cases $s = 0, t < m'$ when $x'_1 \leq x_1$ resp. $t = 0, s < m'$ when $x'_{m'} \geq y_m$ (*i. e.*, start points of some "inbetween" errands are outside of the forward pass).
- Use sum arrays for x'_i and y'_i to compute the extra distance for the inbetween part in $\mathcal{O}(1)$.

H – Hectic Harbour



H – Hectic Harbour

Problem

Schedule two gantry cranes such that they finish their assigned tasks as fast as possible.

H – Hectic Harbour

Solution

- Define two DP arrays for cranes A and B:

  dpA[i][j][p]: A finished task i, B finished task j.
  A is at position of task i, B is at position p.

  dpB[i][j][p]: A finished task i, B finished task j.
  A is at position p, B is at position of task j.

- Distinguish three cases when updating DP arrays:
  1. A and B both perform their next task if they need exactly the same number of steps. If not, consider case (2) or (3).
  2. A performs next task while B moves as close as possible towards its next task.
  3. B performs next task while A moves as close as possible towards its next task.
- Always ensure that cranes do not crash.
- Complexity: $\mathcal{O}(a\,b\,n)$
- Sweep line solutions in $\mathcal{O}(a\,b\,n\log(n))$ are also accepted.

Weiteres Programm

- Jetzt: Auflösung des Scoreboards und Siegerehrung
- Anschließend Voice-Chat auf dem Discord-Server
- Extended Contest mit den GCPC-Aufgaben (bald) unter

`https://domjudge.cs.fau.de/`

Danke für die Teilnahme!